

# Comment on “Random-Field Spin Model beyond 1 Loop: A Mechanism for Decreasing the Lower Critical Dimension”

In a recent letter [1], several interesting results in the random field  $O(N)$  spin model near four dimensions are obtained by a two-loop functional renormalization group. The existence of nonanalytic fixed points with a linear cusp is one of them, which is shown by a large  $N$  analysis with the renormalization group. It is argued that several of these fixed points are once or twice unstable and they yield a long crossover or the metastability of the system within a glassy region. In this comment, however, we indicate that these nonanalytic fixed points are unphysical. Here, we present our understanding of the phase transition in this model for large  $N$ .

In  $d = 4 + \epsilon > 4$  dimensions, the random field spin model has a random anisotropy function  $R(z)$  as an effective interaction. As discussed in the letter [1], the fixed point condition in the large  $N$  limit is expressed as

$$-\tilde{R}(z) + 2\tilde{R}'(1)\tilde{R}(z) - \tilde{R}'(1)\tilde{R}'(z)z + \frac{1}{2}\tilde{R}'(z)^2 = 0, \quad (1)$$

up to two-loop order in terms of a rescaled random anisotropy function  $\tilde{R}(z) = \lim_{N \rightarrow \infty} NR(z)/\epsilon$ . Denoting  $y(z) = \tilde{R}'(z)$  and  $y_0 = y(1)$ , a family of the fixed points can be parameterized by an integer  $n \geq 2$ , such that

$$y_0 = \frac{n}{n-1}, \quad z = y - (y_0 - 1) \left( \frac{y}{y_0} \right)^n. \quad (2)$$

An even  $n$  corresponds to the random field fixed point, and a stability analysis based on the eigenvalue problem of the scaling operator indicates that it has  $n$  relevant modes [2]. An odd  $n$  corresponds to the random anisotropy fixed point, and it has  $(n-1)/2$  relevant modes. In the random field systems, however, the correlation function exponents should satisfy the Schwartz-Soffer inequality [3,4]  $\bar{\eta} \leq 2\eta$ . This inequality is obtained by a simple argument on the correlation functions of the random field systems. The formulae for the correlation exponents [4]

$$\eta = R'(1), \quad \bar{\eta} = (N-1)R'(1) - \epsilon \quad (3)$$

and the Schwartz-Soffer inequality imply  $R'(1) \leq \frac{\epsilon}{N-3}$ , which is valid for any finite  $N$ . Thus,  $y_0 \leq 1$  in the large  $N$  limit, if the corresponding fixed point is physical for finite  $N$ . Therefore the nonanalytic fixed points with  $y_0 = \frac{n}{n-1} > 1$ , for finite  $n = 2, 3, 4, \dots$  cannot give any useful information for large but finite  $N$ . As long as the model has any small random field, this upper bound gives useful restriction on the model. If one discusses a model with a special constraint  $R(-z) = R(z)$  to forbid the random field, one has no upper bound. Here, we do not discuss such a random anisotropy model. Therefore, the only physical fixed points are the trivial fixed point

$y(z) = 0$ , the dimensional reduction fixed point  $y(z) = 1$  and the second rank random anisotropy fixed point  $y(z) = z$ , in the large  $N$  limit.

The stability analysis indicates that  $y(z) = 0$  is fully stable,  $y(z) = 1$  is once unstable and  $y(z) = z$  is fully unstable [2]. For large, but finite  $N$ , the fixed points with sufficiently large  $n$  may satisfy the Schwartz-Soffer inequality. However, they are unstable as well as the fixed point  $y(z) = z$ . On the other hand at the fixed point  $y(z) = 1$ , infinitely many relevant modes recognized as serious instabilities during these two decades are understood as a weak singularity of a nonanalytic fixed point by Tarjus and Tisser (TT fixed point)[5]. This fixed point is the unique once unstable fixed point given by  $R'_{\text{TT}}(z) = R'_{\text{DR}}(z) + a(1-z)^\alpha + \dots$ , where  $\alpha = \frac{N}{2} - \frac{9}{2} + \dots$  and  $R_{\text{DR}}(z)$  is analytic at  $z = 1$  and  $R'_{\text{DR}}(1) = \frac{\epsilon}{N-2}$ . The finiteness of  $R_{\text{TT}}(z)$  at  $z = -1$  should fix the coefficient  $a$ . Although neither a simple large  $N$  expansion nor critical exponents can distinguish the TT fixed point from the dimensional reduction, our stability analysis works to specify the TT fixed point.

For a sufficiently weak randomness, the renormalization group flow is absorbed into the fully stable fixed point  $R(z) = 0$  universally. This fixed point characterizes the ferromagnetic phase. The universal properties of the phase transition between the ferromagnetic and disordered phases are classified into the following two cases. In the first case for  $N \geq 18 - \frac{49}{5}\epsilon$ , the flow goes into the unique once unstable TT fixed point at the critical strength of the random field. The dimensional reduction is observed in the critical exponents  $\eta = \bar{\eta} = \frac{\epsilon}{N-2}$ . The unique relevant mode is a finite eigenfunction with the eigenvalue  $\epsilon + \frac{\epsilon^2}{N} + \frac{2\epsilon^2}{N^2} + \dots$ , which confirms another prediction  $\frac{1}{\nu} = \epsilon + \frac{\epsilon^2}{N-2} + O(\epsilon^3)$  by the dimensional reduction. The amplification of this relevant mode leads to the disordered phase. In the second case for  $N < 18 - \frac{49}{5}\epsilon$ , the TT fixed point disappears. It is believed that the phase transition is controlled by a nonanalytic fixed point with a linear cusp. The critical exponents  $\eta$  and  $\bar{\eta}$  are shifted from the values of the dimensional reduction [4].

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[1] P. Le Doussal and K. J. Wiese, Phys. Rev. Lett. **96**, 197202 (2006).

[2] Y. Sakamoto, H. Mukaida and C. Itoi, Phys. Rev. B **72**, 144405 (2005); *ibid* **74**, 064402, (2006).

[3] M. Schwartz and A. Soffer, Phys. Rev. Lett. **55**, 2499 (1985).

[4] D. E. Feldman, Phys. Rev. B **61**, 382 (2000); Phys. Rev. Lett. **88**, 177202 (2002).

[5] Tarjus and Tisser, Phys. Rev. Lett. **93**, 267008 (2004).